J. Fluid Mech. (1972), *vol.* 52, *part* 2, *pp.* 269–272
Printed in Great Britain

Note on the stability of a boundary layer on a concave wall with suction

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(Received 8 April 1971 and in revised form 3 September 1971)

The purpose of this paper is to consider theoretically how the homogeneous suction from a slightly concave permeable wall will affect the instability of the laminar boundary layer to the onset of longitudinal vortices. The curves of neutral stability and of several growth factors of the vortices are given. The critical values of the Görtler parameter G_c and the wavenumber σ_c, based on the momentum thickness of the boundary layer, are found: $G_c = 1{\cdot}17$ and $\sigma_c = 0{\cdot}22$.

1. Introduction

It is known that the increase in the critical Reynolds number by suction for a laminar boundary layer along a flat plate comes mainly from the sensible dependence of the instability to two-dimensional wavelike disturbances (so-called Tollmien–Schlichting instabilities) on the basic velocity profile in the laminar boundary layer. When a laminar boundary layer becomes unstable on a slightly concave region of the boundary surface, another instability to three-dimensional disturbances, resulting in the onset of Taylor–Görtler vortices, should also be considered. In the case without suction, the numerical results of Görtler (1940) and Hämmerlin (1955) show that the curve of neutral stability depends insensibly upon the basic velocity profile in the laminar boundary layer. It may therefore be supposed that variation of the basic velocity profile due to the suction does not become a factor which changes the critical value of the Görtler parameter as a characteristic value noticeably. It is nevertheless interesting to study how the normal component of velocity, which is induced in the boundary layer by the suction, will contribute to the instability under consideration.

2. Stability diagram

The equations governing the linear stability problem are the same as those for the no-suction problem (see Hämmerlin 1955, p. 284) except that the terms $(-v_0\,\delta/\nu)\,u'$ and $(-v_0\,\delta/\nu)\,(v''' - \sigma^2 v')$, which arise from suction, appear in the left-hand side of the first and the second of his equations (2.5), respectively. Figure 1 is a stability diagram with solid lines showing a typical case of an asymptotic suction profile $U/U_\infty = 1-\exp(0{\cdot}5y/\theta)$, which appears far enough downstream from a laminar boundary-layer flow along a permeable flat plate (and also a slightly concave wall in the first-order approximation of the boundary layer) with homogeneous suction velocity v_0. For all v_0, the suction Reynolds number $-v_0\theta/\nu$ becomes 0·5. Here U_∞ denotes a free-stream velocity, U a flow velocity at a normal co-ordinate y, which is measured from the concave wall, θ the momentum thickness of the boundary layer and ν the kinematic viscosity of the fluid. Görtler's parameter G is here defined as $(U_\infty\,\theta/\nu)(\theta/R_0)^{\frac{1}{2}}$, the non-dimensional wavenumber σ as $\alpha\theta$ and the growth factor B as $\beta\theta^2/\nu$, where R_0 is the radius of curvature at the concave wall, α the wavenumber of disturbances and β a measure of the rate of growth of the disturbances with time.

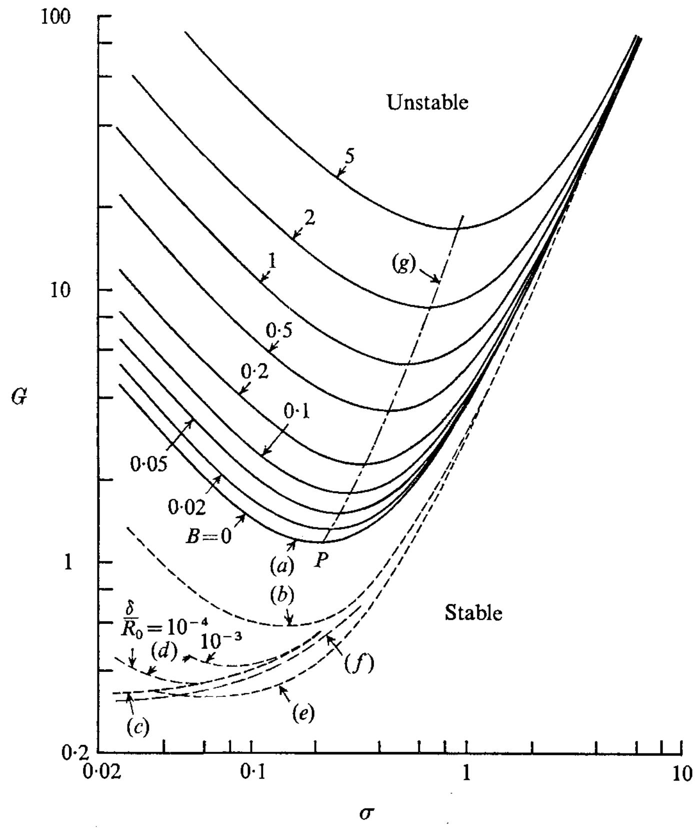

FIGURE 1. Stability diagram of relation between the Görtler parameter G and the wavenumber σ. ———, the case of the homogeneous suction; (a) the curve of neutral stability; P, the critical point; (g) the curve drawn through the minimum point of the Görtler parameter for each growth factor B. – – –, the case without suction; (b) Görtler; (c), (f) Hämmerlin; (d) Witting; (e) Smith.

In figure 1 the curve (a) for $B = 0$ gives the state of neutral stability, which is compared with the previous results (shown as the curves (b), (c), (d) and (e)) in the case without suction. The curve (b) was first calculated by Görtler (1940), the curves (c) and (d) respectively by Hämmerlin (1955) and Witting (1958) with higher approximation, and the curve (e) by Smith (1955), who considered the streamwise growth of a laminar boundary layer. The basic velocity distribution used in the calculations for (b)–(e) was the Blasius profile for a flat plate. Hämmerlin (1955) also calculated a neutral curve (f) (see figure 1), using the asymptotic suction profile as a basic velocity distribution, but in the case without suction. His aim was to study the dependence of the curve of neutral stability on the basic velocity profile. Figure 1 indicates that the boundary layer is stabilized by the homogeneous suction over the whole range of the wavenumber σ. The effect of the suction becomes more remarkable as σ decreases. As is known from the distributions of perturbation velocities, which were obtained as characteristic functions in the course of our numerical calculation, the longitudinal vortices appear closer to the wall in the present case than those in the case without suction, so that viscous effects near the centre of the vortices become stronger. This follows the increase in the parameter G in the neutral state, the trend being more remarkable for small values of σ.

The critical state, defined as the minimum point of the parameter G on the curve of neutral stability, has the critical value $G_c = 1{\cdot}17$ at $\sigma_c = 0{\cdot}22$ in the present case. The corresponding critical value obtained by Witting in the case without suction was $G_c = 0{\cdot}36$–$0{\cdot}41$ for $\delta/R_0 = 10^{-4}$–10^{-3}, where δ is the thickness of the boundary layer; Smith gives $G_c = 0{\cdot}32$. It might be therefore said that the critical value G_c of the Görtler parameter increases by a factor of about three owing to the homogeneous suction. The increase in the critical value should not be attributed to the variation of the basic velocity profile, but definitely to the normal velocity induced in the laminar boundary layer by the suction. It should further be noticed that the increase in the critical Görtler parameter remains much less than that in the critical Reynolds number for Tollmien–Schlichting instability due to the homogeneous suction. This suggests that Görtler instability will so predominate in the laminar boundary layer along the concave wall with suction that the transition point will be determined by this type of instability.

We shall therefore consider the wavelength ($\lambda = 2\pi/\alpha$) of the longitudinal vortices. By eliminating the thickness θ of the boundary layer from the definition of the parameter G and the wavenumber σ, we obtain a relation

$$G^2/\sigma^3 = U_\infty^2 \lambda^3/(8\pi^3\nu^2 R_0) \equiv C.$$

If the velocity U_∞ of the main flow and the radius R_0 of the curvature at the wall, as well as the wavelength λ of the longitudinal vortices, remain constant in the direction of the main flow, C becomes a constant. Experiments in the case without suction by Tani (1962) and by McCormack, Welker & Kelleher (1970) indicate that the longitudinal vortices grow in the streamwise direction with constant wavelength after their onset. The growth of the vortices with a constant wavelength can be shown by a straight line of gradient $\frac{3}{2}$ in figure 1 since both axes of figure 1 are on a logarithmic scale. When the constant C has a value at which the straight line is below the curve (a) of neutral stability, the vortices with the wavelength λ corresponding to the constant C are damped with the time, owing to viscous effects. When $C \geqslant 13{\cdot}1$ the straight line touches or cuts the neutral curve (a), and the boundary layer enters an unstable region; the possible range of wavelengths becomes

$$\lambda \geqslant kR_0(U_\infty R_0/\nu)^{-\frac{2}{3}},$$

where $k = 14.8$. The value of k in the case without suction was given as $13{\cdot}6$ by Görtler and is $12{\cdot}3$ when calculated from Smith's results. The present value of k, which gives the lower limit of the possible wavelength, is respectively 9 and 20% larger than Görtler's and Smith's results in the case without suction. If the Görtler parameter G increases from the critical value G_c by a factor of three along the curve (a) of neutral stability, the constant k varies in the range 14·8–467, which corresponds to a change 32 times as large in the value of the wavelength λ. It means that the selectivity of the wavelength at the appearance of the longitudinal vortices is weak.

The most possible wavelength, λ_c say, may be that of the critical point P, at which k is 31·8. The corresponding value of k in the case without suction was 42–60 (for $\delta/R_0 = 10^{-3}$–10^{-4}) when calculated from Witting's results, and 42 when obtained from Smith's results. The critical wavelength λ_c in the case where the homogeneous suction is present is therefore shorter than that in the case without suction. As the critical wavenumber σ_c is 0·22, the value λ_c is 28θ, where θ is the momentum thickness of the laminar boundary layer, or 3δ, where the thickness δ is defined at the value of y where $U(y) = 0{\cdot}99U_\infty$.

The curve (g) in figure 1 is drawn from the critical point P through the minimum point of the parameter G for each growth factor B. The gradient of the curve (g) for small values of B is nearly equal to $\frac{3}{2}$. It might therefore be expected that the most possible process is that the longitudinal vortices appear at the critical point P and then grow in the streamwise direction with a constant wavelength.

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